Ordinary Isotropic Peridynamic Models Position Aware Viscoelastic(PAV́E) SAND2015-3078 C

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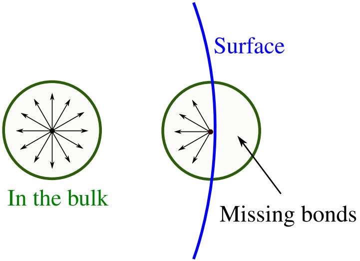
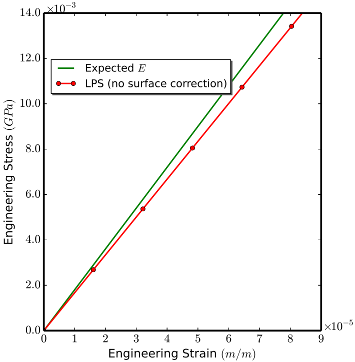
Ordinary peridynamic models: surface effects
Position Aware models correct for this

Causes relate to material points near surface

- Mathematical models assume all points are in the *bulk*
  - Points near surface are *missing bonds*
  - *Missing bonds* imply and induce incorrect material properties
  - In the bulk mathematical models are consistent
- Kinematic defects at the surface

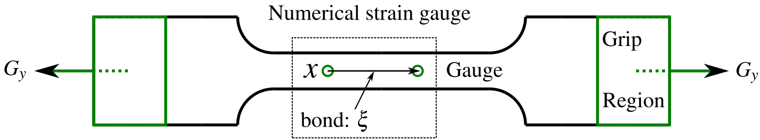


Surface effects in ordinary peridynamic models
Tension test: ordinary isotropic elastic model (LPS)



The following related aspects contribute to mismatch.

- Geometric surface effects
- Nonlocal model kinematics
- Nonlocal model properties
- Discretization error



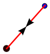
Position Aware Models
Outline

- Review the practical issue/problem of surface effects
- Introduce *Position Aware* models
- Selecting/creating/evaluating influence functions (briefly)
- Demonstration calculations
  - Position Aware Linear Solid (PALS)
  - Position Aware Viscoelastic (PAV́E)

Maturation & extension of material models

Bond-based

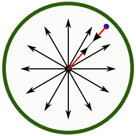
– micro-brittle (PMB)
– plasticity



State-based

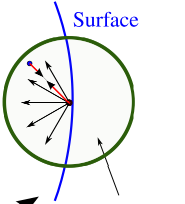
– linear solid (LPS)
– plastic
– viscoelastic

extended



Position-aware

– linear solid (PALS)
– viscoelastic (PAV́E)



generalized

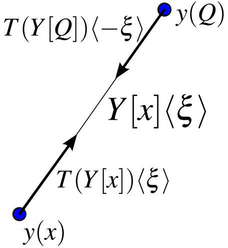
generalized

Ordinary material models
Silling, Epton, Weckner, Xu, and Askari, 2007

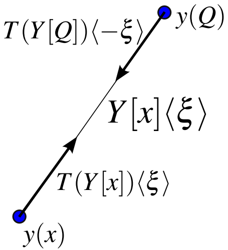
Integral equation for internal force density f of particle x

$$
\begin{aligned}
\rho(x)\ddot{u}(x,t) &= f(x,u(x,t),t) + b(x,t) \\
f(x,u(x,t),t) &= \int_H \{T(Y)[x]\langle\xi\rangle - T(Y)[Q]\langle -\xi\rangle\}\, dV_Q
\end{aligned}
$$

Ordinary



Ordinary material models
Silling, Epton, Weckner, Xu, and Askari, 2007

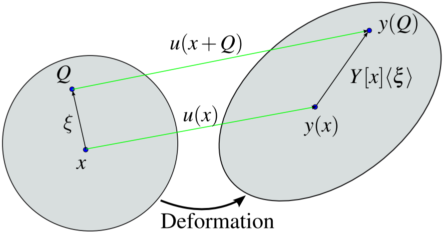


The vector force state T is given as:

$$T(Y) = t(Y)M(Y) \quad \text{where} \quad M(Y) = \frac{Y}{|Y|}$$

Scalar force state $t(Y)$ defines *ordinary* material model. More later.

Kinematic peridynamic states: $\underline{e}$, θ, $\underline{\varepsilon}$



Scalar extension state: $\underline{e}$

$$\underline{e}\langle\xi\rangle = |Y| - |\xi|$$

Dilatation: θ

$$\begin{aligned}\theta &= (\underline{\omega}|\xi|)\bullet\underline{e}\\ &= \int_H \underline{\omega}|\xi|\underline{e}\langle\xi\rangle\, dV_Q\end{aligned}$$

Deviatoric extension state: $\underline{\varepsilon}$

$$\underline{\varepsilon} = \underline{e} - \frac{\theta|\xi|}{3}$$

Position Aware Linear Solid (PALS)
Mitchell, Silling, and Littlewood, 2015

Scalar force state obtained from elastic energy density functional

$$W(\theta, \underline{\varepsilon}) = \frac{\kappa\theta^2}{2} + \mu(\underline{\sigma\varepsilon}) \bullet \underline{\varepsilon}$$

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Scalar force state

$$t(Y) = p\underline{\omega x} + 2\mu\underline{\sigma\varepsilon}$$

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Scalar force state

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Scalar force state with deviatoric *in-elastic* deformations $\underline{\varepsilon}^p$

$$t(Y) = p\underline{\omega x} + 2\mu\underline{\sigma}\underbrace{(\underline{\varepsilon} - \underline{\varepsilon}^p)}_{elastic}$$

Position Aware Viscoelastic(PAVÉ)
Mitchell, 2015

Scalar force state obtained from elastic energy density functional

$$W(\theta, \underline{\varepsilon}) = \frac{\kappa\theta^2}{2} + \mu_\infty(\underline{\sigma}\underline{\varepsilon}) \bullet \underline{\varepsilon} + \sum_i \mu_i(\underline{\varepsilon} - \underline{\varepsilon}^i)\underline{\sigma} \bullet (\underline{\varepsilon} - \underline{\varepsilon}^i)$$

Position Aware Viscoelastic(PAVÉ)
Mitchell, 2015

Scalar force state obtained from elastic energy density functional

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Scalar force state

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Mitchell, 2015

Scalar force state obtained from elastic energy density functional

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Scalar force state

$$\color{blue}{t(Y)} = p\underline{\omega}\underline{x} + 2\mu_\infty\underline{\sigma}\underline{\varepsilon} + 2\sum_i \mu_i\underline{\sigma}(\underline{\varepsilon} - \underline{\varepsilon}^i)$$

Governing equation for $\underline{\varepsilon}^i$

$$\dot{\underline{\varepsilon}}^i + \frac{1}{\tau_i}\underline{\varepsilon}^i = \underline{\varepsilon}(t)$$

PALS: influence function construction

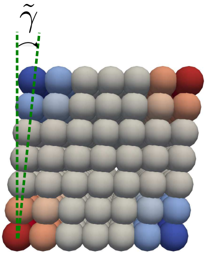
PALS (position aware linear solid) model

- $\underline{\omega}$, $\underline{\sigma}$ are computed for each point in mesh
- Initial influence functions $\underline{\omega}^0$, $\underline{\sigma}^0$ given
- Select $\underline{\omega}$, $\underline{\sigma}$ as best approximations to $\underline{\omega}^0$,$\underline{\sigma}^0$ subject to kinematic constraints: *matching deformations* $\underline{e}^k\langle\xi\rangle = \frac{\xi\cdot H^k\xi}{|\xi|}$

$$I(\underline{\omega},\lambda) = \frac{1}{2}(\underline{\omega}-\underline{\omega}^0)\bullet(\underline{\omega}-\underline{\omega}^0) - \sum_{k=1}^{K}\lambda^k\left[(\underline{\omega x})\bullet\underline{e}^k - \mathrm{Tr}\,\mathbf{H}^k\right]$$

$$N(\underline{\sigma},\tau) = \frac{1}{2}(\underline{\sigma}-\underline{\sigma}^0)\bullet(\underline{\sigma}-\underline{\sigma}^0) - \sum_{k=1}^{K}\tau^k\left[(\underline{\sigma\varepsilon}^k)\bullet\underline{\varepsilon}^k - \gamma^k\right]$$

Model problem: simple shear
PALS versus *LPS*: expectation *dilatation* $\theta = 0$

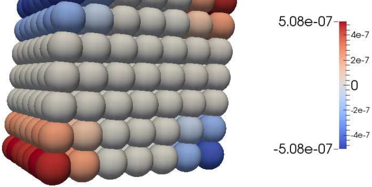


Simple shear

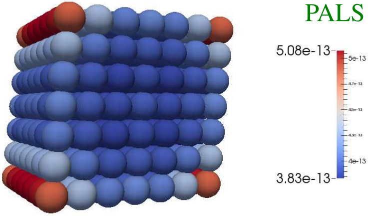
$$u = \tilde{\gamma} y; \quad v = 0; \quad w = 0; \quad \tilde{\gamma} = 1.0 \times 10^{-6}$$

Dilatation

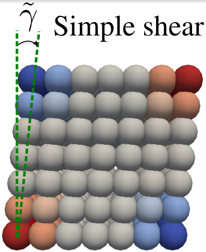
LPS

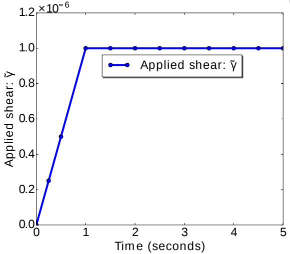


PALS

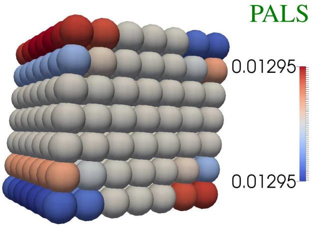


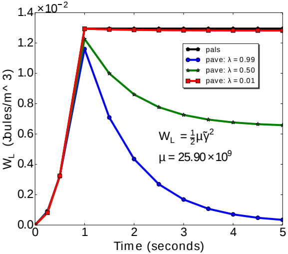
Model problem: simple shear
PALS and *PAVÉ*



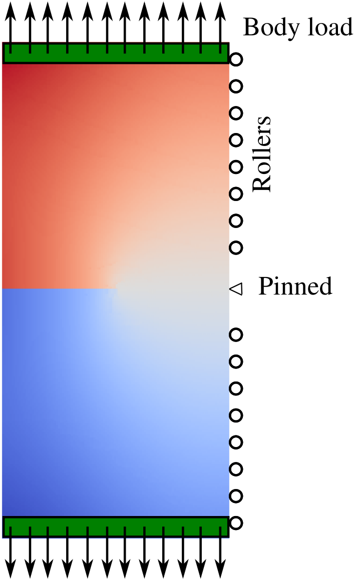


Stored elastic energy density

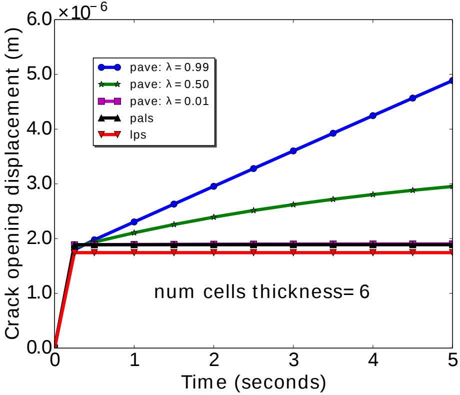


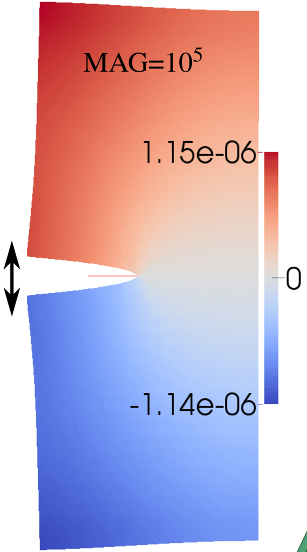


Crack Opening Displacement: Schematic

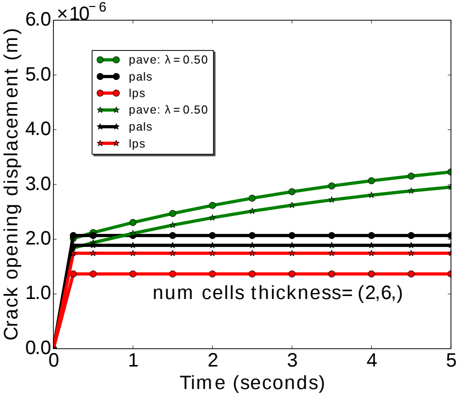


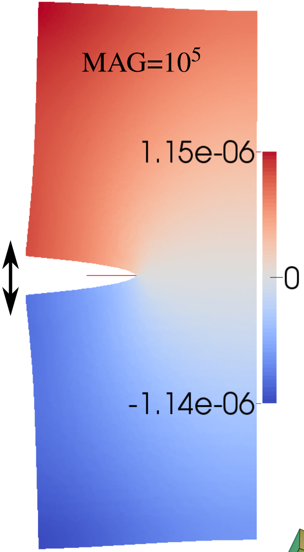
Crack Opening Displacement
Model Convergence



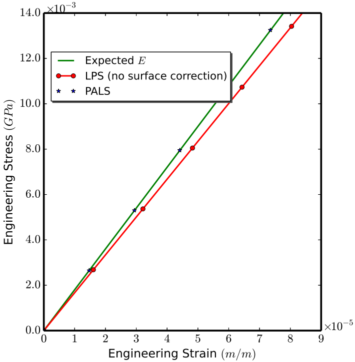


Crack Opening Displacement
Mesh Convergence



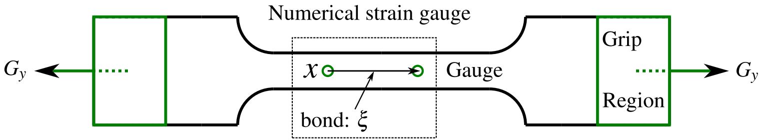


Recover Young's modulus E
Tensile test



PALS model: *sharply reduces surface effects*

PALS model: *significant step toward making peridynamics accurate as a general-purpose simulation capability*



Conclusions

- Reviewed the practical issue/problem of surface effects
- Introduced novel *Position Aware Linear Solid* model (PALS)
  - Addresses inaccuracies (LPS) due to missing bonds near surface
- Introduced novel *Position Aware Viscoelastic* model (PAV́E)
- Demonstration calculations of *new* PAV́E model
- Demonstration calculations show efficacy of PALS

THANK YOU

Questions?